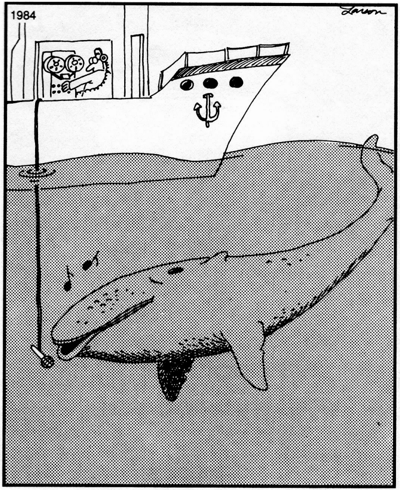
Signal Processing 101

Kathleen E. Wage
George Mason University

Discovery of Sound in the Sea Webinar
12-October-2022



"A Louie, Louie…wowoooo…We gotta go now…"

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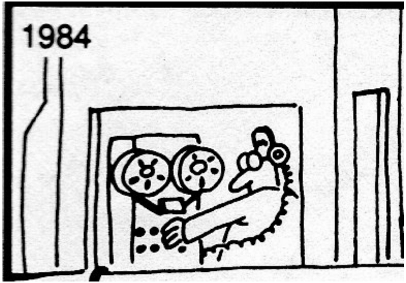
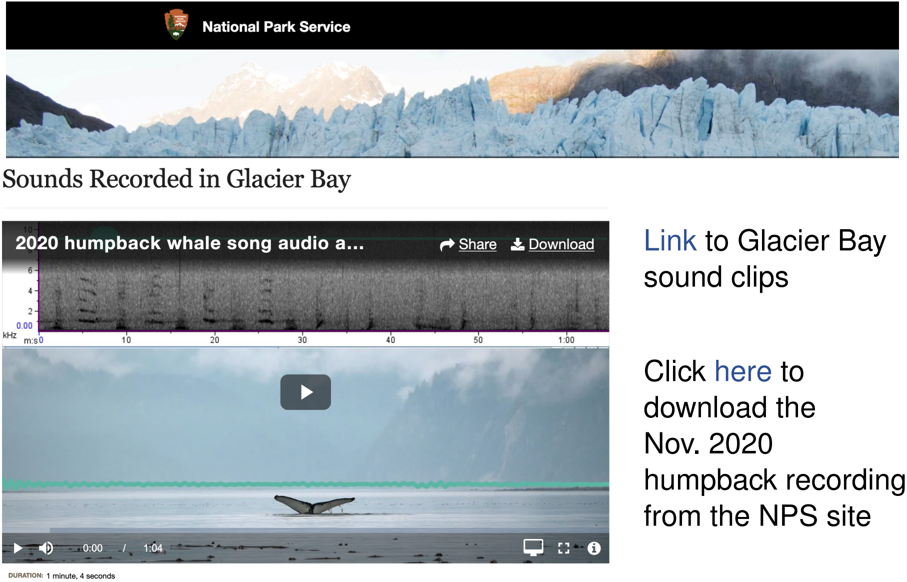


How do we record and analyze signals in 2022?

- How fast to sample?
- How to use a bank of filters to make a spectrogram?

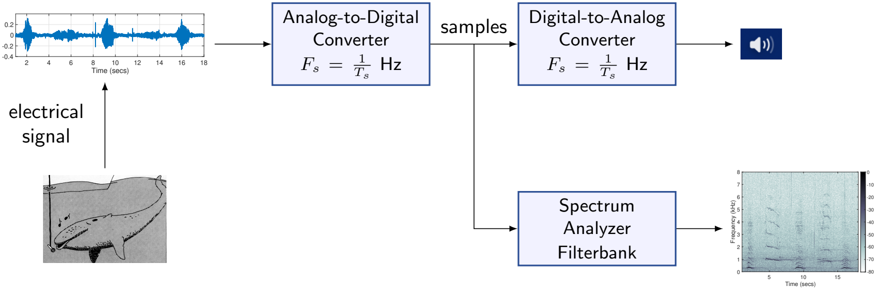
This talk uses a 1-minute recording of a humpback whale from a hydrophone at the mouth of Glacier Bay



Link to Glacier Bay sound clips

Click here to download the Nov. 2020 humpback recording from the NPS site

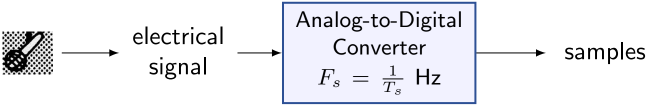
Recording and analysis system consists of three basic components: ADC, DAC, Filterbank

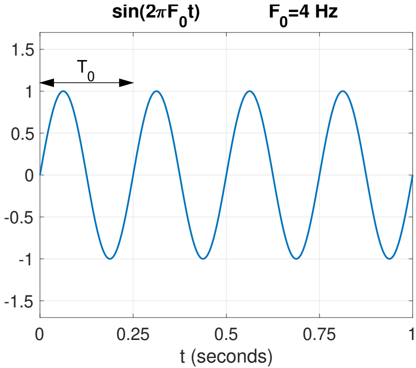


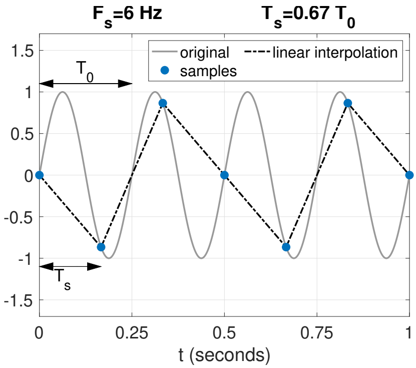
Designing the system requires answering the following questions:

- How fast should we sample the signals? What is F_s?
- How many filters should we use and what determines the resolution of the spectrogram?

Analog-to-Digital Converter (ADC) measures an electrical signal and produces a set of samples







Higher sample rate F_s (smaller sample period T_s) $\longrightarrow$ better plots

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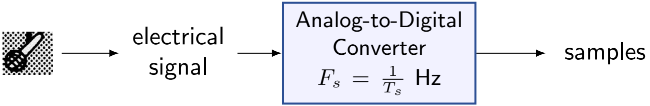

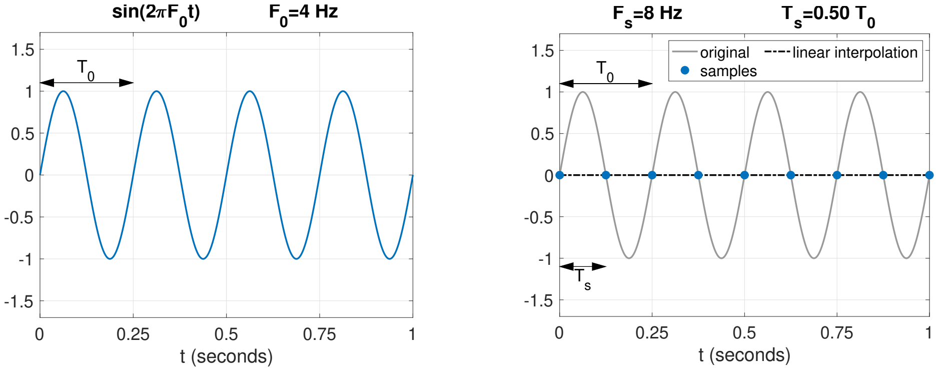

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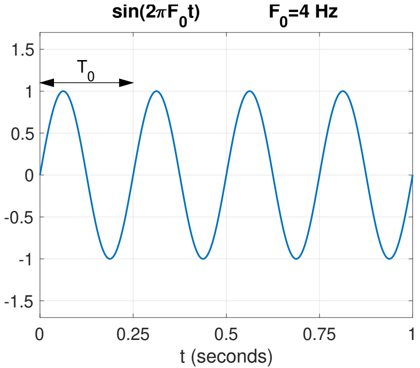

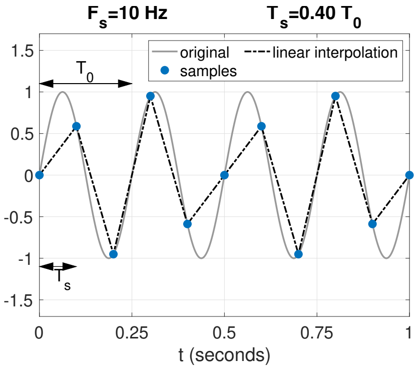

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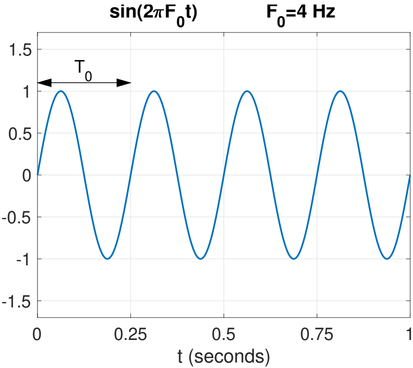

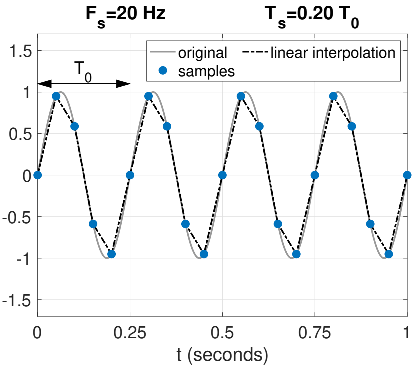

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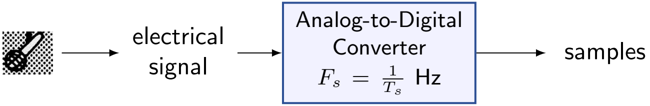

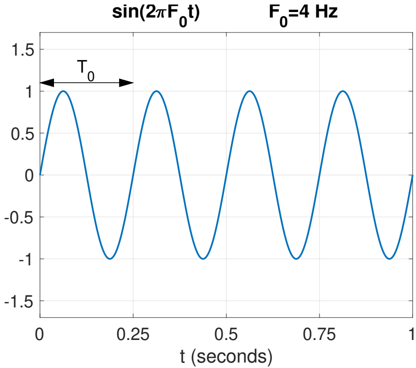

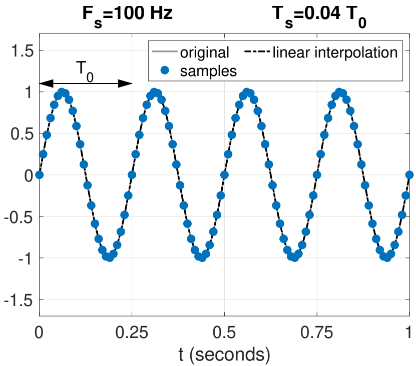

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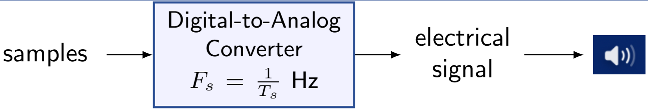
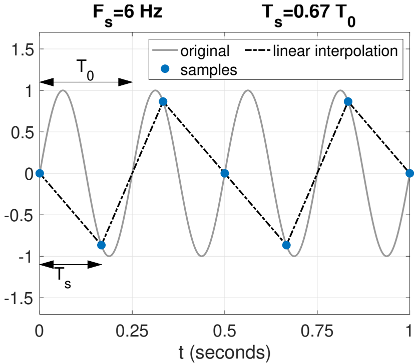
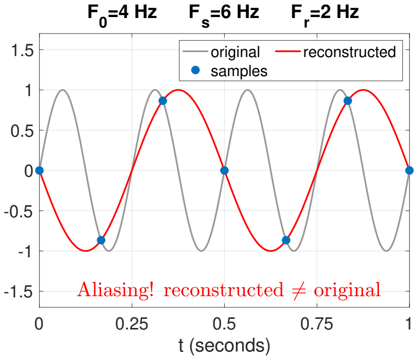
Digital-to-Analog Converter (DAC) reconstructs an electrical signal from a set of samples







Can reconstruct signal if we have more than 2 samples per period

$\longrightarrow T_s < \frac{T_0}{2}$ or $F_s > 2F_0$

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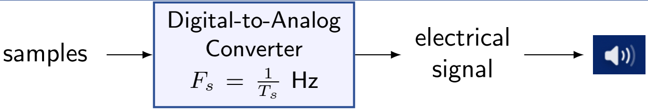

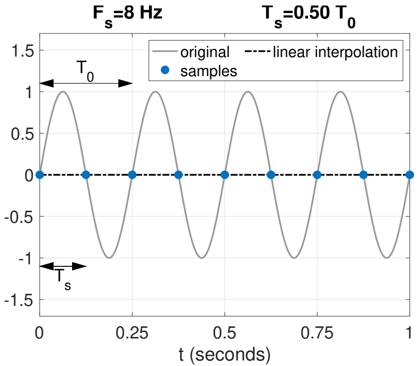

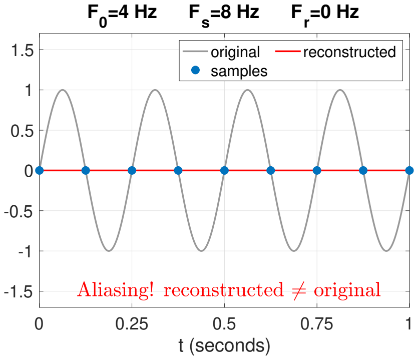

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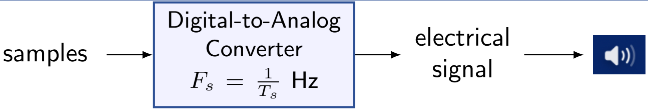

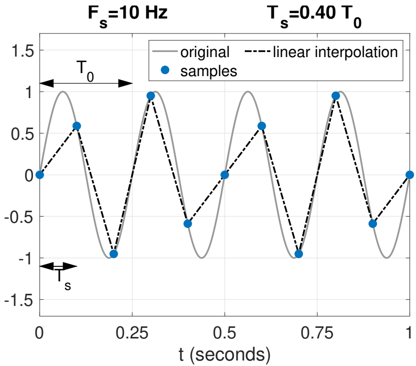

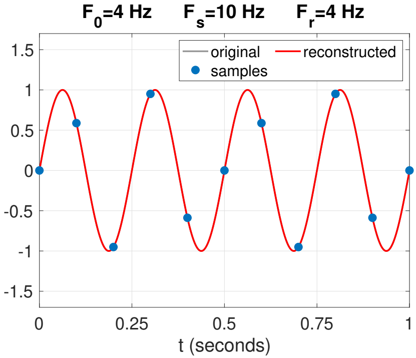

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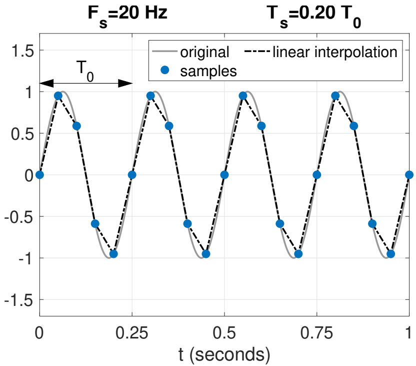
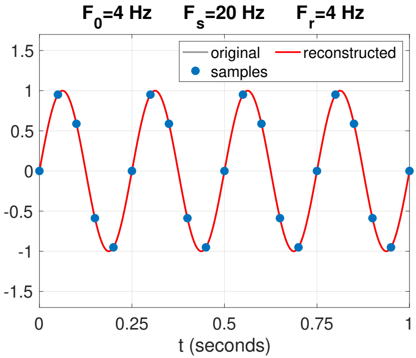
samples → **Digital-to-Analog Converter** $F_s = \frac{1}{T_s}$ Hz → electrical signal →





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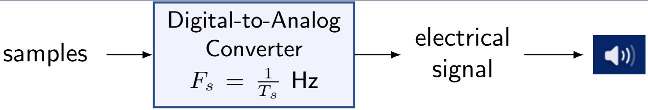

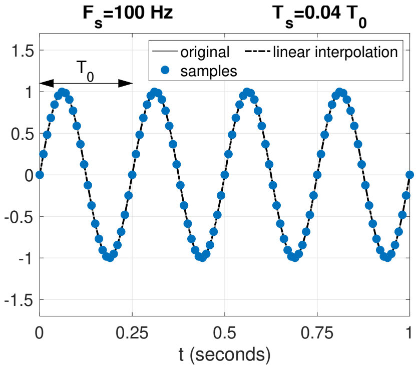

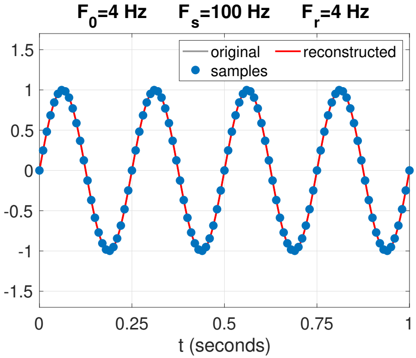

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$\longrightarrow T_s < \frac{T_0}{2}$ or $F_s > 2F_0$

Sampling theorem guarantees that we can reconstruct a signal if we sample it at a high enough rate

<u>Conclusion from the previous examples:</u>

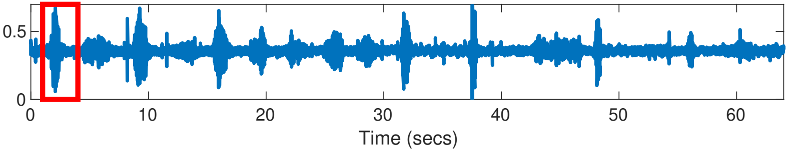
We can reconstruct the signal from its samples if

- $F_s > 2F_0$ or equivalently,
- $T_s < \frac{T_0}{2}$

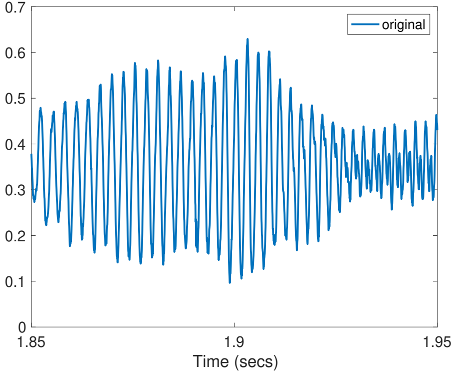
This assumes there is only one sinusoidal signal and we know its frequency (F_0)

What if the whales aren't singing just one note all the time?

We can synthesize a whale signal by adding sinusoids with different frequencies, amplitudes, and phases



$$\text{synth} = \sum_m A_m \cos(2\pi f_m + \phi_m)$$



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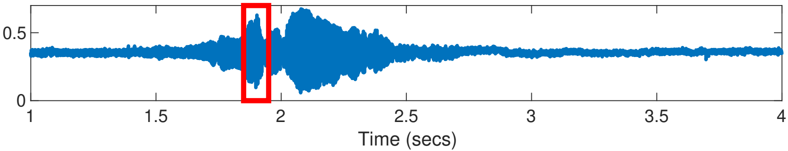

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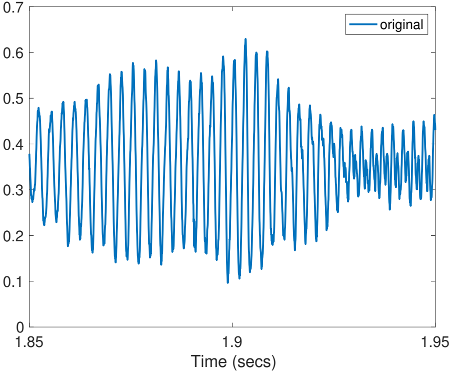

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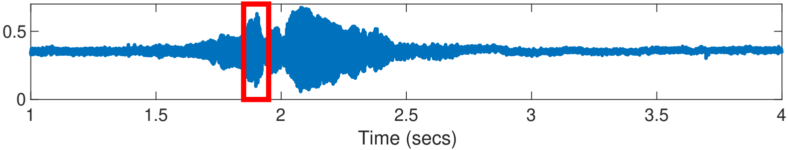

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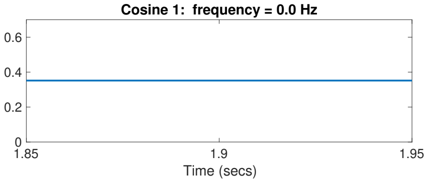

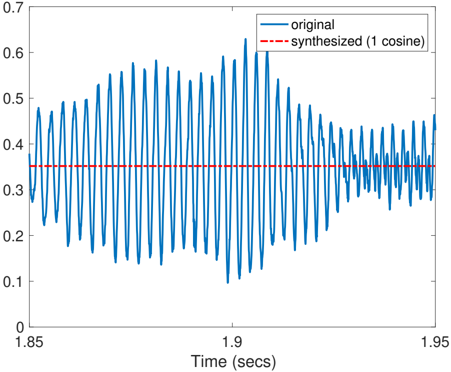

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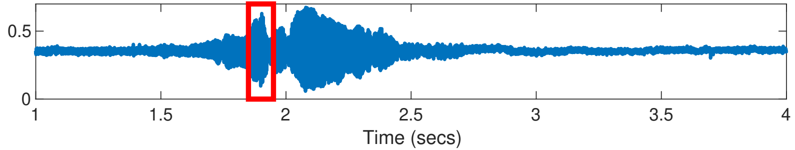

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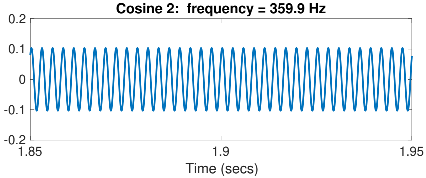

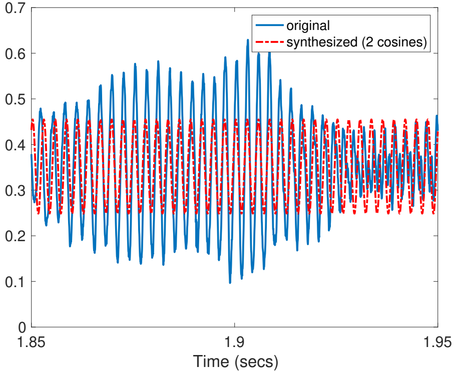

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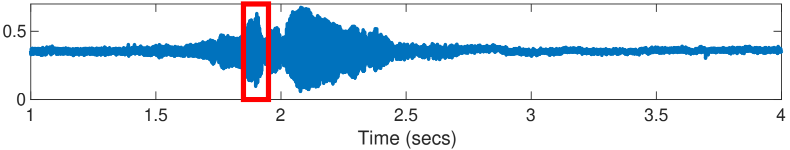

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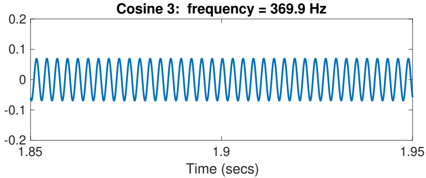

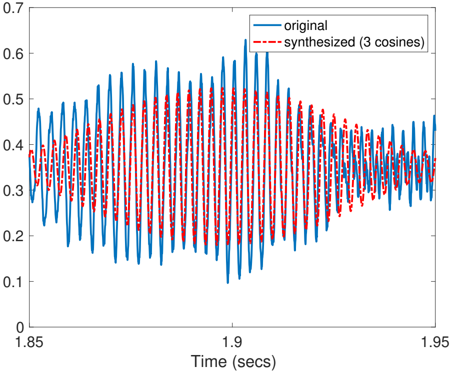

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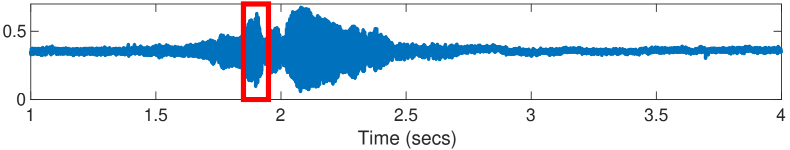

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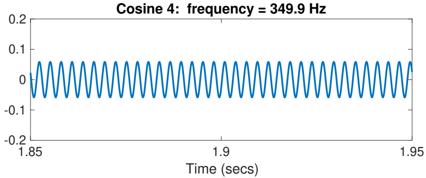

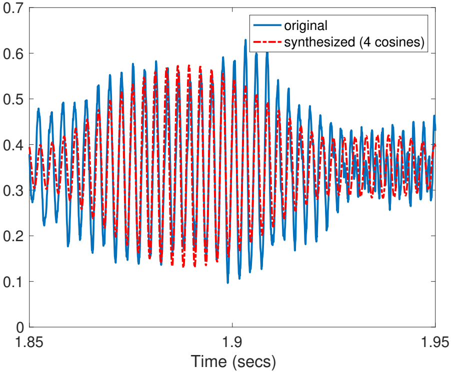

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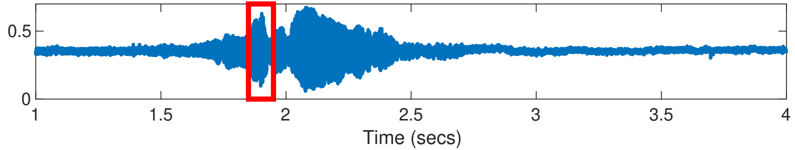

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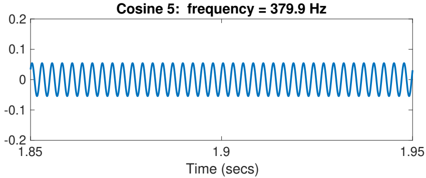

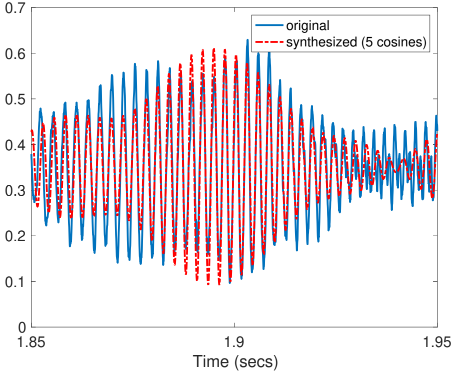

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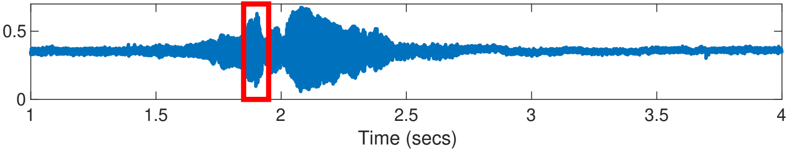

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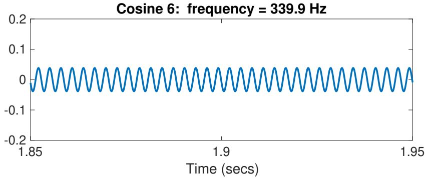

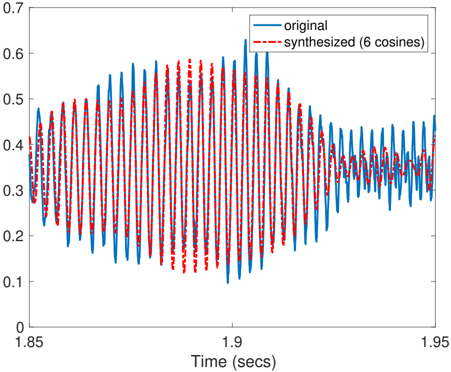

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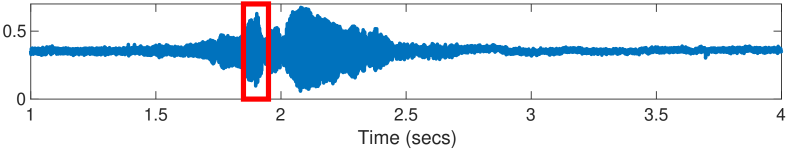

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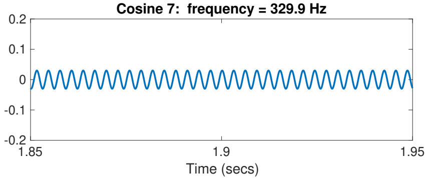

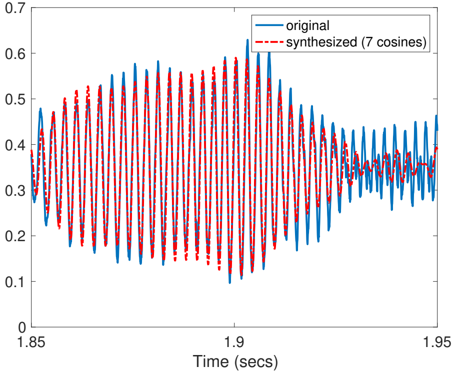

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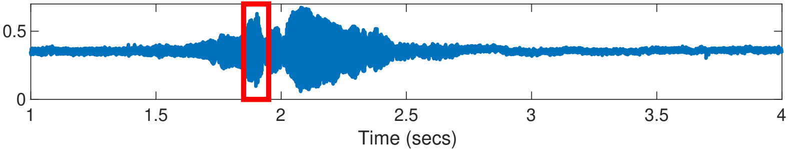

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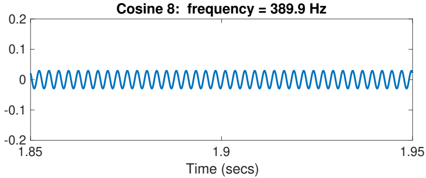

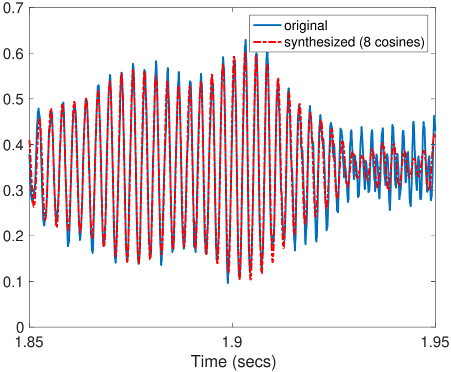

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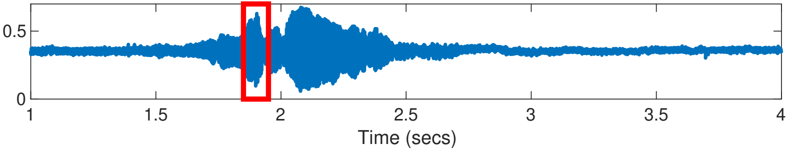

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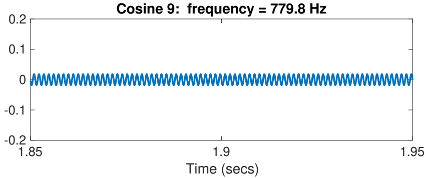

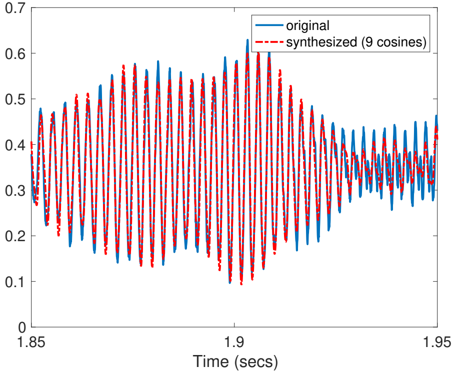

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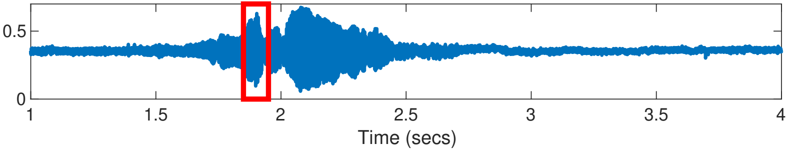

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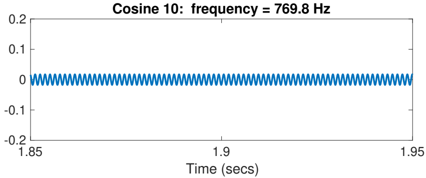
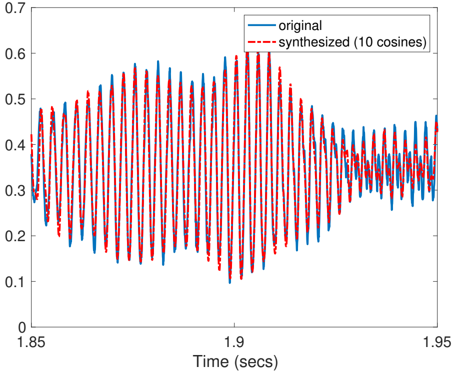




Choose a sample rate $F_s > 2F_{\max}$

Sampling theorem tells us how to choose F_s

To avoid aliasing:

$$F_s > 2F_{\max} \text{ Hz}$$

To make smooth plots from our samples:

$$F_s >> 2F_{\max} \text{ Hz}$$

Use a lowpass anti-alias filter before Analog-to-Digital Converter to remove signals above $\frac{F_s}{2}$ Hz

Sample rate shouldn't be too low (aliasing!) or too high (costly to store and to process!)

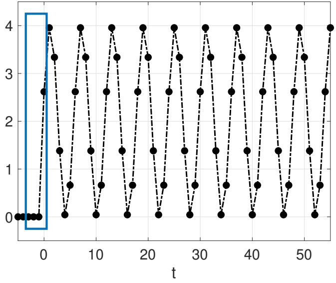
Like Goldilocks, we want F_s to be *just right*

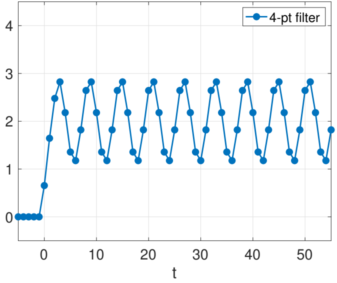
Filter processes samples to enhance signals, e.g., freq.-selective filter removes unwanted frequencies, leaving desired signal alone

samples $x[n]$ → Finite Impulse Response (FIR) Filter → $y[n] = \sum_{l=0}^{L-1} b_l x[n-l]$

Moving average filter: $$y[n] = \frac{1}{L}\sum_{l=0}^{L-1} x[n-l] \qquad b_l = \frac{1}{L}$$

Moving avg is a *lowpass filter* (removes high frequencies)





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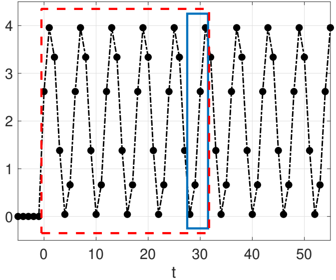

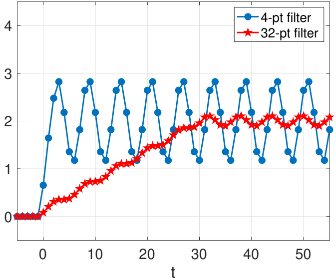

Longer lowpass filters have better rejection

. . . but they take longer to respond to changes in input

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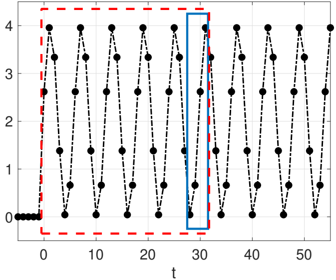

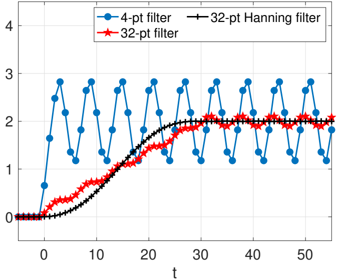

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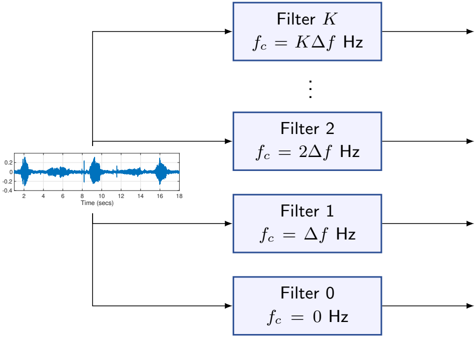
We can design a *bank* of filters that pass different ranges of frequencies



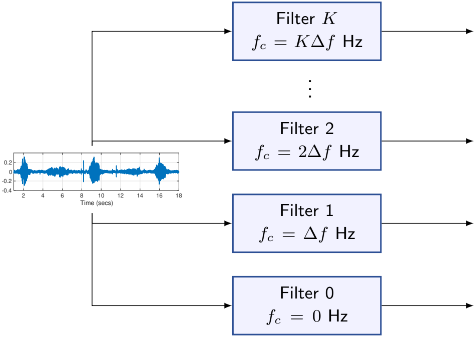
Filterbank design parameters: K and L

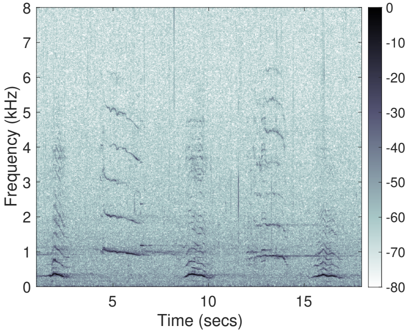
- $K + 1$ filters in the filterbank
- f_c is the center frequency of the filter
- kth filter has $f_c = k\Delta f$
- Resolution determined by the length L of the filter

Longer filters have higher frequency resolution

. . . but they take longer to respond to changes in input

Spectrogram is output of bank of filters

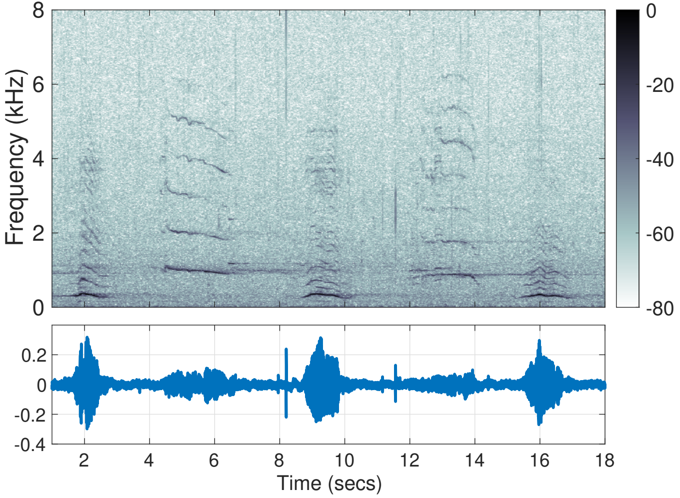




Spectrogram filterbank parameters:

- Length of filter (L) $\longrightarrow$ time window width
- Number of filters ($K + 1$) $\longrightarrow$ size of the transform (FFT size)
- Type of filter $\longrightarrow$ window used (e.g., Hanning)

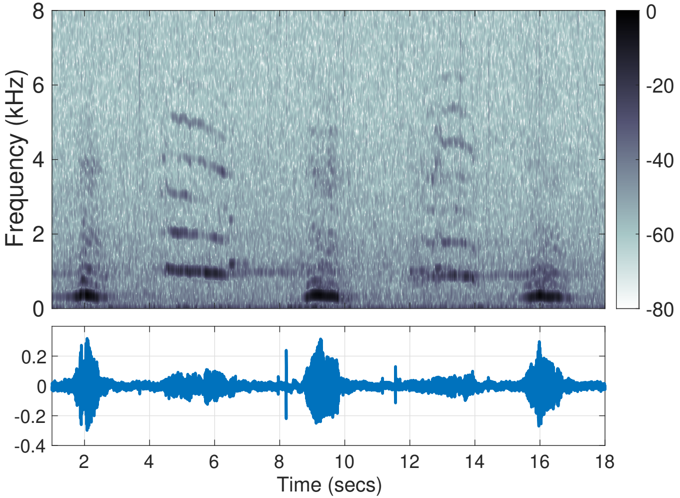
Spectrogram of the humpback whale signal displays its frequency content



Filter length = 0.10 secs

Longer filters have higher frequency resolution and lower time resolution

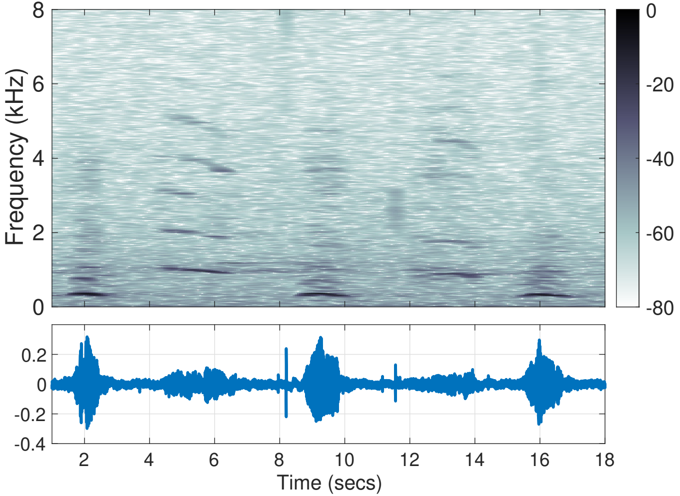
Spectrogram of the humpback whale signal displays its frequency content



Filter length = 0.01 secs

Longer filters have higher frequency resolution and lower time resolution

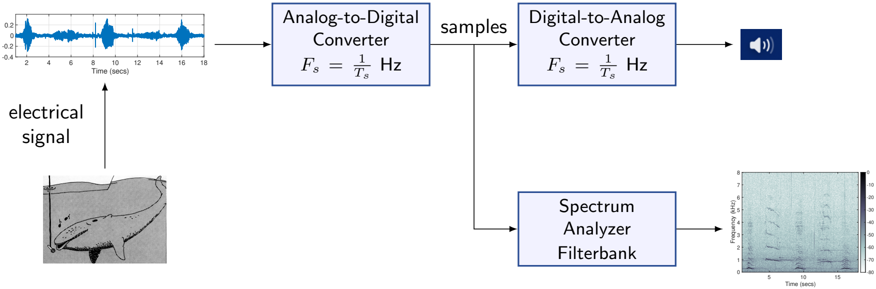
Spectrogram of the humpback whale signal displays its frequency content



Filter length = 1.00 secs

Longer filters have higher frequency resolution and lower time resolution

Recording and analysis system consists of three basic components: ADC, DAC, Filterbank



Designing the system requires answering the following questions:

- How fast should we sample the signals?
  $F_s > 2F_{\mathrm{max}}$ Hz
- What determines the resolution of the spectrogram?
  Length of the filter (time window) determines the time/frequency resolution of the spectrogram

For more information

- Watch some videos, e.g.,
  https://www.youtube.com/ProfKathleenWage
- Talk to students who are taking signal processing
- Take a class: signals and systems or signal processing (electrical/computer engineering, ocean engineering)
- Read a book, e.g., *DSP First 2nd Edition* by McClellan, Schafer, Yoder
  https://dspfirst.gatech.edu
- Consult an expert signal processor